A Time-Truncated Life Test Approach on the new Two-sided Group Chain Sampling Plan with Log-logistic Distribution

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***Abstract*— The established new two-sided complete group chain sampling plan (NTSCoGCh), which operates with five acceptance criteria, and the two-sided group chain sampling plan (TS-GCh), which operates with three acceptance criteria, are consumer- and producer-focused plans respectively. Since the number of acceptance criteria influences the responding probability of lot acceptance in lot inspection, this paper suggests the balanced approach of the new two-sided group chain sampling plan (NTSGCh) which operates with four acceptance criteria. The lifetime distribution of Log-logistic is used in this study, and the method of time truncated life test simulation is conducted. The findings proved that the NTSGCh performed better than the NTSCoGCh. Therefore, the NTSGCh is a worthy option for implementation in lot inspection in the manufacturing industry.**

***Keywords*—** *acceptance sampling, Log-logistic distribution, new two-sided group chain sampling plan, probability of lot acceptance, truncated life test.*

1. Introduction

Acceptance sampling is a commonly-practiced quality control approach in manufacturing, more specifically in the inspection process. It enables the producer to decide on whether to accept or reject a production lot without having to test and inspect each and every one of the products within that lot. It is a more cost-effective approach as compared to total inspection or 100% inspection which involves inspecting every product. Over the years, there had been many researchers suggesting new technique or sampling plans in acceptance sampling, often with different combinations of sampling plans and lifetime distributions. Some examples of these techniques are the single sampling plan with the Exponential distribution [1], the double sampling plan with the Rayleigh distribution [2], the chain sampling plan with the Weibull distribution [3], and the group sampling plan with the Weibull distribution [4]. The underlying reason for these different lifetime distributions being used is simply for exploration. Different products are being manufactured which follows different sets of lifetime distributions. Hence, the reason for applying a sampling plan to different lifetime distributions is to observe the behaviour of that sampling plan when it is used for different products.

Moreover, there are also extensions of established plans that were introduced over the years. These plans are similar to the established plans, with changes made to the technical aspect of the plan. For example, the modified chain sampling plan by Govindaraju and Lai [5] is an extension of the chain sampling plan, which is proven to perform better than the chain sampling plan for production lots with poor quality standard. Not only that, some researchers identified areas of improvement between two sampling plans and suggested a technique which involves combining the two plans. For example, ref. [6] suggested combining the group chain sampling plan by ref. [7] and the modified chain sampling plan by ref. [5] and naming the new plan as the modified group chain sampling plan. The modified group chain sampling plan is proven to overcome the shortcomings of both its parents'.

2. The Family of Two-sided Group Chain Sampling Plan

Ref. [8] first introduced the two-sided approach in chain sampling. This sampling plan is known as the two-sided complete chain sampling plan. As opposed to the chain sampling method of checking the historical performance of the production process to make decision, the two-sided complete chain sampling plan requires the producer to analyze the historical and future performance of the process prior to lot sentencing. In other words, the decision of accepting or rejecting the preceding and succeeding lots heavily influence the decision to accept or reject the current lot in inspection. The benefit of this plan is that it demands a continuously high-quality background from the producer.

Then, ref. [9] saw the potential of combining the two-sided complete chain sampling plan and the group chain sampling plan. The plan was named the new two-sided complete group chain sampling plan (NTSCoGCh). Similar to the plan by Deva and Rebecca [8], the NTSCoGCh considers the results of inspection from both the preceding and succeeding lots prior to current lot sentencing, with the additional ability to conduct multiple inspections simultaneously thanks to the grouping mechanism from the group chain sampling plan.

Another similarity shared between the NTSCoGCh and the two-sided complete group chain sampling plan lies in the technical aspect of the acceptance criteria. The acceptance criteria are a series of potential outcomes from the inspection of the preceding, current and succeeding lots that are deemed acceptable for further processing. In other words, if the outcome of inspection of the three lots obey the pre-specified acceptance criteria, then the current tested lot will be accepted. If not, then it will be rejected. Both NTSCoGCh and the two-sided complete chain sampling plan operates on five acceptance criteria, out of eight possible criteria from the inspection. Hence, both sampling plans implied the term ‘complete’ in their names.

Later, ref. [10] came up with another plan within the family tree of the two-sided group chain sampling plan. The plan, simply named as the two-sided group chain sampling plan (TS-GCh), operates with only three acceptance criteria. This ultimately changed the operational procedure of the plan due to it being much stricter in accepting the tested lot. Mughal [10] found that although the probability of lot acceptance for the TS-GCh experienced a slight decrease as compared to the NTSCoGCh, it requires a smaller sample size. Thus, it is still more cost-effective in comparison to the NTSCoGCh, at the slight expense of probability of lot acceptance.

From a theoretical point of view, the number of acceptance criteria only guides the sampling plan towards its purpose. A high number of acceptance criteria makes the plan more than likely to be accepted, hence it can be said that it is a producer-focused plan. That is, there is an increasing risk of a type-II error, the error in accepting a lot with a big portion of defective products in it [11]. On the other hand, a sampling plan with a low number of acceptance criteria is less likely to be accepted, hence, in cases where the production quality of the producer is poor, then it can be said that it is a consumer-focused sampling plan. Similarly, in such cases, there are chances of a type-I error occurring. Type-I error is explained by the act of rejecting a lot with a big portion of functioning or non-defective products in it [11].

Therefore, this paper intends to propose a new sampling plan. The plan will be named as the new two-sided group chain sampling plan (NTSGCh). It will operate with four acceptance criteria, hence bridging the gap in the consumer-producer focused plans. Not only that, it will also be interesting to observe how the plan will perform in contrast to the legacy sampling plans, NTSCoGCh and TS-GCh.

3. Methodology

3.1 Time truncated life test

A time truncated life test is useful in inspecting products with respectively long lifetime. The products are set to work starting at time $t = 0$ and ending at time $t = t0 = b\mu0$. In this study, the pre-specified termination time, t_0, is a multiple of the specified mean lifetime of the product, $\mu0$, and the specified time constant, b. Then, the number of defectives found at $t0$ is tallied. This method saves a lot of time as opposed to observing the time taken for the products to stop functioning.

3.2 Operational Procedure

The step-by-step operational procedure for the NTSGCh is listed below:

I. Draw a sample of size n and divide it into g groups of r testers. Then, start the life test.

II. Stop the test at $t = t0$. Inspect all units simultaneously and count the number of defectives, d.

III. If $d > 1$, reject the production lot.

IV. If $d = 0$, accept the production lot given that the preceding and succeeding lots have at most 1 defective unit, $d_i + d_j \leq 1$.

V. If $d = 1$, accept the lot if and only if the cumulative number of defectives in the preceding and succeeding lots is zero, $d_i + d_j = 0$.

3.3 Proportion defective

The proportion defective is derived from the cumulative distribution function (CDF) of the lifetime distribution involved. In this study, the Log-logistic distribution is used. The CDF for the Log-logistic distribution is:

$$F(t;\delta,\gamma) = \frac{(\frac{t}{\delta})^{\gamma}}{1+(\frac{t}{\delta})^{\gamma}} \tag{1}$$

The true mean lifetime for products following the Log-logistic distribution is given by:

$$\mu = \frac{\pi\delta/\gamma}{\sin(\pi/\gamma)} \tag{2}$$

Thus, by simplifying equations (1) and (2), the proportion defective for the Log-logistic distribution is:

$$p = \frac{(1.5708b)^{2}}{\left(\frac{\mu}{\mu_0}\right)^{2} + (1.5708b)^{2}} \tag{3}$$

3.4 Probability of lot acceptance

The acceptance number is set as 1 for the purpose of this test. In other words, finding more than 1 defective in a lot under inspection will result in immediate rejection of that lot. Thus, it is critical that the inspected lot yields either 0 or 1 defective unit for it to be accepted. P_0 and P_1 denotes the probabilities of finding 0 and 1 defectives respectively in the lot under inspection. Since the NTSGCh considers the same amount of preceding and succeeding lots, which means $i = j$, the probabilities of finding 0 and 1 defectives in the preceding and succeeding lots are denoted by P_0^i and P_1^i respectively. Hence, the four acceptable outcomes that makes up the four acceptance criteria can be simplified into:

$$P(a) = P_0^{2i}[(2i+1)P_1 + P_0] \tag{4}$$

Due to the dichotomous nature in lot sentencing, equation (4) can be further derived with Binomial properties to obtain the following equation of probability of lot acceptance:

$$P(a) = [(1-p)^{(gr)}]^{2i} \cdot [(2i+1)\cdot grp(1-p)^{gr-1} + (1-p)^{gr}]$$
(5)

The design parameters in the study are specified at multiple values to observe the behaviour of the NTSGCh across different parameter settings. These values are r = {2,3,4,5}, $i = j$ = {1,2,3,4}, b = {0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00}, β = {0.10, 0.05, 0.01} and $\mu/\mu0$ = {1,2,4,6,8,10,12}.

4. Results and Discussion

4.1 The minimum number of groups

One of the performance indicators when comparing sampling plans is the minimum number of groups. The sampling plans with the grouping mechanism would require the drawn sample to be broken down into a minimum number of groups, with each group consisting a certain number of testers. Ultimately, a lower minimum number of groups means that there are lesser testers needed to inspect the products simultaneously. Hence, it would lessen the operating costs of inspection. Table 1 below lists the minimum number of groups yielded from the truncated life test simulation of the NTSGCh.

Table 1. The minimum number of groups for the NTSGCh using Log-logistic distribution

			b							
β	*r*	*i*	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
0.1	2	1	5	2	1	1	1	1	1	1
	3	2	2	1	1	1	1	1	1	1
	4	3	1	1	1	1	1	1	1	1
	5	4	1	1	1	1	1	1	1	1
0.05	2	1	6	2	2	1	1	1	1	1
	3	2	3	1	1	1	1	1	1	1
	4	3	2	1	1	1	1	1	1	1
	5	4	1	1	1	1	1	1	1	1
0.01	2	1	8	3	2	1	1	1	1	1
	3	2	4	1	1	1	1	1	1	1
	4	3	2	1	1	1	1	1	1	1
	5	4	2	1	1	1	1	1	1	1

The information from Table 1 can easily be interpreted using the formula for the breaking down of the sample into groups, where $n = g$ x r. For example, let's say a semiconductor manufacturer wants to inspect a production lot. The company operates with 10% consumer's risk (β = 0.1) and assigned two employees for the inspection process. Let's say that the life test to be conducted will only be done at 25% of the specified mean lifetime of their products. Thus, the minimum number of groups is 5, hence the minimum number of samples needed is 10.

4.2 The probability of lot acceptance

Another performance indicator being used when comparing sampling plans is the probability of lot acceptance. It refers to the probability that the production lot in testing will adhere to one of the four acceptance criteria thus being accepted for further processing. Table 2 below lists the yielded probability of lot acceptance for the NTSGCh.

Table 2. The probability of lot acceptance for NTSGCh using the Log-logistic distribution

			$\mu/\mu 0$						
β	*g*	*b*	1	2	4	6	8	10	12
0.1	5	0.25	0.0761	0.6933	0.9668	0.9927	0.9976	0.9990	0.9995
	2	0.50	0.0263	0.5099	0.9289	0.9833	0.9944	0.9976	0.9988
	1	0.75	0.0503	0.5160	0.9230	0.9813	0.9936	0.9973	0.9987
	1	1.00	0.0091	0.2631	0.8143	0.9481	0.9813	0.9918	0.9959
	1	1.25	0.0018	0.1182	0.6697	0.8921	0.9585	0.9813	0.9905
	1	1.50	0.0004	0.0503	0.5160	0.8143	0.9230	0.9640	0.9813
	1	1.75	0.0001	0.0212	0.3764	0.7203	0.8745	0.9388	0.9674
	1	2.00	0.0000	0.0091	0.2631	0.6181	0.8143	0.9052	0.9481
0.05	6	0.25	0.0375	0.6118	0.9537	0.9896	0.9966	0.9986	0.9993
	2	0.50	0.0263	0.5099	0.9289	0.9833	0.9944	0.9976	0.9988
	2	0.75	0.0005	0.1448	0.7521	0.9289	0.9742	0.9887	0.9944
	1	1.00	0.0091	0.2631	0.8143	0.9481	0.9813	0.9918	0.9959
	1	1.25	0.0018	0.1182	0.6697	0.8921	0.9585	0.9813	0.9905
	1	1.50	0.0004	0.0503	0.5160	0.8143	0.9230	0.9640	0.9813
	1	1.75	0.0001	0.0212	0.3764	0.7203	0.8745	0.9388	0.9674
	1	2.00	0.0000	0.0091	0.2631	0.6181	0.8143	0.9052	0.9481
0.01	8	0.25	0.0086	0.4638	0.9229	0.9820	0.9939	0.9974	0.9988
	3	0.50	0.0021	0.2857	0.8574	0.9637	0.9874	0.9946	0.9973
	2	0.75	0.0005	0.1448	0.7521	0.9289	0.9742	0.9887	0.9944
	1	1.00	0.0091	0.2631	0.8143	0.9481	0.9813	0.9918	0.9959
	1	1.25	0.0018	0.1182	0.6697	0.8921	0.9585	0.9813	0.9905
	1	1.50	0.0004	0.0503	0.5160	0.8143	0.9230	0.9640	0.9813
	1	1.75	0.0001	0.0212	0.3764	0.7203	0.8745	0.9388	0.9674
	1	2.00	0.0000	0.0091	0.2631	0.6181	0.8143	0.9052	0.9481

A mean ratio, $\mu/\mu 0$, refers to the ratio of the true mean lifetime to the specified average lifetime of the products. Thus, a positive integer means that the products can last longer than it is meant to last. Also, a higher value of mean ratio means that the general quality of the products produced is better in contrast to products produced from a process with lower mean ratio. This can be observed in Table 2, where the probability of lot acceptance increases as the mean ratio increases.

Furthermore, the effect of the life test duration can also be seen in Table 2. It is evident that as the specified time constant, b, increases, the resulting probability of lot acceptance decrease. Since the test time is the multiple of b and $\mu 0$, a high value of b could drag the life test closer to the failure time of the products in inspection. Thus, it is more likely to find defective products during a longer life test, compared to one with a shorter life test.

To further illustrate, consider the company in the previous example. The company produces 1000 semiconductors in a production lot. It is estimated that the specified mean lifetime of the semiconductors is 1000 hours. The company is only willing to spend 25% of the specified mean lifetime for the inspection, hence b = 0.25. Thus, the design parameters for the inspection are (β, r, i, g, b) = (0.1, 2, 1, 5, 025). Hence, the company will find that the probability of lot acceptance will be in the range of 0.0761 to 0.9995, depending on the mean ratio of their process.

4.3 Comparison with the NTSCoGCh

In this section, the performance of the NTSGCh and the NTSCoGCh by [9] are compared. Another simulation was conducted, this time with the algorithm of the NTSCoGCh while using the Log-logistic distribution. The comparison in the minimum number of groups can be observed in Table 3 below. The difference in the two sampling plans are especially distinctive when the parameters are (β, r, i) = (0.05, 2, 1).

Table 3. The comparison in the minimum number of groups

b	*NTSGCh*	*NTSCoGCh*
0.25	6	7
0.50	2	2
0.75	2	2
1.00	1	1
1.25	1	1
1.50	1	1
1.75	1	1
2.00	1	1

The NTSGCh requires a lower minimum number of groups as compared to the NTSCoGCh. Hence, it can be said that the minimum sample size is also lower in comparison. Ultimately, in terms of costs, inspection with the NTSGCh will require a lower operational cost.

It is also interesting to see how the two sampling plans differ when it comes to the resulting probability of lot acceptance. Table 4 below shows the comparison of the probability of lot acceptance of the two plans.

Table 4. The comparison in probability of lot acceptance

$\mu/\mu 0$	g	
	NTSGCh	NTSCoGCh
	6	7
1	0.0375	0.0294
2	0.6118	0.5942
4	0.9537	0.9511
6	0.9896	0.9890
8	0.9966	0.9964
10	0.9986	0.9985
12	0.9993	0.9993

Not only does the NTSGCh operates at a lower cost, it also seems to be performing better when it comes to inspecting production lots with lower mean ratio. As evident in Table 4, the NTSGCh yields a comparatively higher probability of lot acceptance at mean ratios 1, 2, 4 and 6. Bearing in mind that this result is obtained when both sampling plans are adhering to 5% consumer's risk. Therefore, at the same level of risk, NTSGCh performed better overall in contrast to NTSCoGCh.

5. Conclusion

This paper suggests a new sampling plan derived from the family of the two-sided group chain sampling plans. The NTSGCh which operates on the basis of four acceptance criteria is a more balanced approach in contrast to its predecessors the two-sided group chain sampling plan and the new two-sided complete group chain sampling plan. It has been proven that the NTSGCh operates at a lower cost compared to the NTSCoGCh while being able to produce a higher probability of lot acceptance. Thus, it can be said that the NTSGCh is a good alternative for the inspection process for producers in the industry.

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References

[1] Epstein, B., "*Truncated life tests in the exponential case*", Annals of Math. Stat., vol. 25, pp. 555-564, 1954.

[2] M. Aslam, "*Double Acceptance Sampling based on Truncated Life Tests in Rayleigh Distribution*," Eur. J. Sci. Res., vol. 17, no. 4, pp. 605-606, 2007.

[3] A. R. S. Ramaswamy & S. Jayasri, "*Time Truncated Chain Sampling Plan for Weibull Distribution*," Int. J. Eng. Res. Gen. Sci., vol. 3, no. 2, pp. 59-67, 2015.

[4] M. Aslam & C. H. Jun, "*A Group Acceptance Sampling Plan for Truncated Life Test Having Weibull Distribution*," J. Appl. Stat., vol. 36, no. 9, pp. 1021-1027, 2009.

[5] K. Govindaraju & C.D. Lai, "*A modified ChSP-1 chain sampling plan, M ChSP-1, with very small sample sizes*," American J. Math. and Mgmt. Sci., vol. 18, pp. 346-358, 1998.

[6] A.F. Jamaludin, Z. Zain, & N. Aziz, "*A modified group chain sampling plans for lifetimes following a Rayleigh distribution*," Global J. Pure and Appl. Maths., vol. 12, no. 5, pp. 3941-3947, 2016.

[7] A.R. Mughal, Z. Zain, & N. Aziz, "*Time truncated Group Chain Sampling Strategy for Pareto Distribution of the 2nd Kind*." Res. J. Appl. Sci., Eng. and Tech., vol. 10, no. 4, pp. 471-474, 2015.

[8] A.S. Deva, & I.E.K. Rebecca, "*Two sided complete chain sampling plans for attribute quality characteristics (CChSP-0,1)*", Karunya J. of Res., vol. 3, no. 1, pp. 8-16, 2012.

[9] A.R. Mughal, Z. Zain, & N. Aziz, "*New two sided complete group chain sampling plan for Pareto distribution of the 2nd Kind*", Res. India Publ., vol. 10, no. 12, pp. 31855-31860, 2015.

[10] A.R. Mughal, "*A family of group chain acceptance sampling plans based on truncated life test*", PhD thesis, Universiti Utara Malaysia, 2018.

[11] D.C. Montgomery, "*Introduction to Statistical Quality Control*", John Wiley & Sons, 2009.